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Dynamic Panel Data Models Featuring Endogenous Interaction and Spatially Correlated Errors*

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Abstract

We extend the three-step generalized methods of moments (GMM) approach of Kapoor, Kelejian, and Prucha (2007), which corrects for spatially correlated errors in static panel data models, by introducing a spatial lag and a one-period lag of the dependent variable as additional explanatory variables. Combining the extended Kapoor, Kelejian, and Prucha (2007) approach with the dynamic panel data model GMM estimators of Arellano and Bond (1991) and Blundell and Bond (1998) and supplementing the dynamic instruments by lagged and weighted exogenous variables as suggested by Kelejian and Robinson (1993) yields new spatial dynamic panel data estimators. The performance of these spatial dynamic panel data estimators is investigated by means of Monte Carlo simulations. We show that differences in bias as well as root mean squared error between spatial GMM estimates and corresponding GMM estimates in which spatial error correlation is ignored are small.

JEL codes: C15, C21, C22, C23

Keywords: Dynamic panel models, spatial lag, spatial error, GMM estimation

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1 Introduction

The separate literatures on dynamic panel data models and spatial econometric models have matured rapidly and have reached (graduate) textbooks during the last decade.[1] Panel data may feature state dependence—that is, the dependent variable is correlated over time—as well as display spatial dependence, that is, the dependent variable is correlated in space. Applied economists' interest in frameworks that integrate spatial considerations into dynamic panel data models is a fairly recent development, however.[2] Elhorst (2003, 2008a,b) and Yu, De Jong, and Lee (2008) have analyzed the properties of maximum likelihood (ML) estimators and combinations of ML and corrected least squares dummy variables (CLSDV) for this model class. Recently, the flexible generalized methods of moments (GMM) framework for dynamic panels has gained popularity.[3] The properties of spatial GMM estimators have not been comprehensively studied in a dynamic panel data context yet.[4] This paper therefore compares the performance of various spatial GMM estimators of dynamic panel data models with fixed effects.

Spatial panel data applications typically employ either a spatial lag model or a spatial error model. Many economic interactions among agents are characterized by a spatially lagged dependent variable, which consists of observations on the dependent variable in other locations than the "home" location. In the public finance literature, for example, local governments take into account the behavior of neighboring governments in setting their tax rates (cf. Wilson, 1999; and Brueckner, 2003) and deciding on the provision of public goods (cf. Case, Rosen, and Hines, 1993). In the trade literature, foreign direct

[1]See Arellano (2003) and Baltagi (2008, Chapter 8) for an analysis of dynamic panel data models and Anselin (1988, 2006) for a treatment of spatial econometrics.

[2]Badinger, Mueller, and Tondl (2004) and Jacobs, Ligthart, and Vrijburg (2009) provide empirical applications of spatial dynamic panel data models.

[3]The GMM framework can handle multiple endogenous explanatory variables, fixed effects, and unbalanced panels.

[4]Elhorst (2008b) is a notable exception, but only briefly touches upon difference GMM estimator with endogenous interaction effects with a view to compare them to spatial ML estimators.

investment (FDI) inflows into the host country depend on FDI inflows into proximate host countries (cf. Blonigen et al., 2007). The spatial lag structure allows one to explicitly measure the strength of the spatial interaction. Spatial error dependence is an alternative way of capturing spatial aspects. It may arise, for example, in house price models in which air quality affects house prices but is not included as an explanatory variable (cf. Kim, Phipps, and Anselin, 2003).[5] Spatially correlated errors can be thought of as analogous to the well-known practice of clustering error terms by groups, which are defined based on some direct observable characteristic of the group. In spatial econometrics, the groups are based on spatial "similarity," which is typically captured by some geographic characteristic (e.g., proximity). Spatial error correlation has not been studied before in dynamic panels.[6]

Recently, Kapoor, Kelejian, and Prucha (2007) designed a GMM procedure to deal with spatial error correlation in static panels. We extend their three-step spatial procedure to panels with a spatially lagged dependent variable *and* a one-period time lag of the dependent variable. The performance of the spatial GMM estimators[7]—which is measured in terms of bias and root mean squared error (RMSE)—is investigated by means of Monte Carlo simulations. We consider panels with a small number of time periods relative to the number of units. Rather than modeling either a spatial error or a spatial lag model, we allow both processes to be present simultaneously. In economic interaction models, spatial error dependence may exist above and beyond the theoretically motivated spatial lag structure,[8] reflecting the potential presence of omitted spatial variables. Ignoring spatial

[5]Spatial error correlation may also result from measurement error in variables or a misspecified functional form of the regression equation.

[6]After having written the first draft of our paper, we learned about the work of Kukenova and Monteiro (2009), who also deal with spatial dynamic panel models based on system GMM (which is based on Blundell and Bond (1998), see Section 3.2). Their framework, however, does not correct for spatial error correlation and incorporates another endogenous explanatory variable in addition to the spatial lag and time lag of the dependent variable.

[7]The term "spatial" GMM estimators refers to GMM estimators for panel data models including a spatial lag with or without correction for spatial error correlation. If the spatial GMM estimator corrects for spatial error correlation, we speak of "spatially corrected" GMM estimators.

[8]In their study on commodity tax competition, Egger, Pfaffermayr, and Winner (2005) find a signifi-

error correlation in static panel data models may give rise to a loss in efficiency of the estimates and may thus erroneously suggest that strategic interaction is absent. In contrast, disregarding spatial dependency in the dependent variable comes at a relatively high cost because it gives rise to biased estimates (cf. LeSage and Pace, 2009, p. 158). This paper takes up these issues in a dynamic panel context.

The time lag of the (endogenous) dependent variable is correlated with the unit-specific effect. Consequently, the standard fixed effects estimator for (non-spatial) panels with a fixed time span and a large number of units is biased and inconsistent. Dynamic panel data models are usually estimated using the GMM estimator of Arellano and Bond (1991), which differs from static panel GMM estimators in the set of moment conditions and the matrix of instruments. The standard Arellano-Bond estimator is known to be rather inefficient when instruments are weak (e.g., if time dependency is strong) because it makes use of information contained in first differences of variables only. Alternatively, authors have used Blundell and Bond's (1998) system approach, which consists of both first-differenced and level equations and an extended set of instruments. In the following, we contribute to the literature by developing spatial variants of the Arellano-Bond and Blundell-Bond estimators. Our new approach involves defining appropriate instruments to control for the endogeneity of the spatial lag and time lag of the dependent variable while controlling for spatial error correlation. To this purpose, we use spatial instruments—which are based on a modification of Kelejian and Robinson (1993)—combined with instruments for dynamic panel data models.

The Monte Carlo evidence indicates that the differences in bias as well as RMSE between spatially corrected GMM estimates and corresponding GMM estimates in which spatial error correlation is ignored are small for reasonable parameter values. The average absolute bias and RMSE of the spatial GMM estimators is not affected much by the size of

cantly positive coefficient of the spatial lag while controlling for spatially correlated errors; the spatially autoregressive coefficient is shown to be significantly negative and non-negligible in size.

the spatial lag parameter. Choosing appropriate instruments for both the time and spatial lag is thus sufficient to model the spatial characteristics in our benchmark setting. If the time dimension of the panel rises, techniques to limit the proliferation of instruments are needed. We show that the combination of collapsing the instrument matrix and limiting the lag depth of the dynamic instruments substantially reduces the bias in estimating the spatial lag parameter, but hardly affects its RMSE.

The paper is organized as follows. Section 2 sets out the dynamic spatial panel data model. Section 3 develops the two estimators for spatial dynamic panel data models (i.e., spatially corrected Arellano-Bond and Blundell-Bond estimators) and touches upon econometric issues. Section 4 presents Monte Carlo simulation outcomes. Finally, Section 5 concludes.

2 The Spatial Dynamic Panel Data Model

Consider $i = 1, ..., N$ spatial units and $t = 1, ..., T$ time periods. The focus is on panels with a small number of time periods relative to the number of spatial units. Assume that the data at time t are generated according to the following model:

$$\mathbf{y}(t) = \lambda \mathbf{y}(t-1) + \delta \mathbf{W}_N \mathbf{y}(t) + \mathbf{x}(t)\boldsymbol{\beta} + \mathbf{u}(t), \tag{1}$$

where $\mathbf{y}(t)$ is an $N \times 1$ vector of observations on the dependent variable, $\mathbf{y}(t-1)$ is a one-period time lag of the dependent variable, $\mathbf{W}_N$ is an $N \times N$ matrix of spatial weights, $\mathbf{x}(t)$ is an $N \times K$ matrix of observations on the *strictly exogenous* explanatory variables (where K denotes the number of covariates), and $\mathbf{u}(t)$ is an $N \times 1$ vector of error terms.[9] The scalar parameter λ is the coefficient of the lagged dependent variable, δ is the spatial autoregressive coefficient (which measures the endogenous interaction effect among units),

[9]Our specification does not include $\mathbf{W}_N \mathbf{y}(t-1)$, which yields a so-called spatiotemporal model (see Yu, De Jong, and Lee, 2008). We leave this extension for future research.

and $\boldsymbol{\beta}$ is a $K \times 1$ vector of (fixed) slope coefficients.

The *spatial lag* is denoted by $\mathbf{W}_N \mathbf{y}(t)$, which captures the contemporaneous correlation between unit i's behavior and a weighted sum of the behavior of units $j \neq i$. The elements of $\mathbf{W}_N$ (denoted by w_{ij}) are exogenously given, non-negative, and zero on the diagonal of the matrix. In addition, the elements are row normalized so that each row sums to one. Note that there is little formal guidance on choosing the "correct" spatial weights because many definitions of neighbors are possible. The literature usually employs contiguity (i.e., units having common borders) or physical distance as weighting factors.

The reduced form of Equation (1) amounts to:

$$\mathbf{y}(t) = (\mathbf{I}_N - \delta \mathbf{W}_N)^{-1} \left[\lambda \mathbf{y}(t-1) + \mathbf{x}(t)\boldsymbol{\beta} + \mathbf{u}(t)\right], \tag{2}$$

where

$$(\mathbf{I}_N - \delta \mathbf{W}_N)^{-1} = \mathbf{I}_N + \delta \mathbf{W}_N + \delta^2 \mathbf{W}_N^2 + \delta^3 \mathbf{W}_N^3 + ..., \tag{3}$$

where $\mathbf{I}_N$ is an identity matrix of dimension $N \times N$. Hence, the dependent variable is affected not only by the characteristics of the own spatial unit but also by those of direct "neighbors" and of "neighbors of neighbors." Stationarity of the model not only requires that $|\lambda| < 1$, but also:

$$|\lambda| + \delta\zeta_U < 1 \quad \text{if} \quad \delta \geq 0 \qquad \text{and} \qquad |\lambda| + \delta\zeta_L < 1 \quad \text{if} \quad \delta < 0, \tag{4}$$

where ζ_L and ζ_U denote the smallest (i.e., the most negative) and largest characteristic root of $\mathbf{W}_N$, respectively (cf. Elhorst, 2008a). Because $\mathbf{W}_N$ is (row) normalized, we find $\zeta_U = 1$.[10] Equation (4) yields a tradeoff between the size of λ and δ.

Spatial error correlation may arise when omitted variables follow a spatial pattern, yielding a non-diagonal variance-covariance matrix of the error term $\mathbf{u}(t)$. In the case of

[10] No general results hold for the smallest characteristic root of the matrix of spatial weights. The lower bound ζ_L is typically less than -1. See Elhorst (2008a, p. 422).

spatial correlation, the error structure in Equation (1) is a spatially weighted average of the error components of neighbors, where $\mathbf{M}_N$ is an $N \times N$ matrix of spatial weights (with typical element m_{ij}). More formally, the spatially autoregressive process is given by:

$$\mathbf{u}(t) = \rho \mathbf{M}_N \mathbf{u}(t) + \boldsymbol{\varepsilon}(t), \tag{5}$$

where $\mathbf{M}_N \mathbf{u}(t)$ is the *spatial error* term, ρ is a (second) spatially autoregressive coefficient, and $\boldsymbol{\varepsilon}(t)$ denotes a vector of innovations. The interpretation of the "nuisance" parameter ρ is very different from δ in the spatial lag model, in that there is no particular relation to a substantive theoretical underpinning of the spatial interaction. We follow the common practice in the literature by assuming that $\mathbf{W}_N \neq \mathbf{M}_N$.[11] The spatial error process in reduced form is: $\mathbf{u}(t) = (\mathbf{I}_N - \rho \mathbf{M}_N)^{-1} \boldsymbol{\varepsilon}(t) = \boldsymbol{\varepsilon}(t) + \rho \mathbf{M}_N \boldsymbol{\varepsilon}(t) + \rho^2 \mathbf{M}_N^2 \boldsymbol{\varepsilon}(t) + \rho^3 \mathbf{M}_N^3 \boldsymbol{\varepsilon}(t) + \ldots$. Shocks in the spatial error representation have a global effect. Intuitively, a shock in location k directly affects the error term of location k but also indirectly transmits to other locations (with a non-zero m_{ij}) and eventually works its way back to k. If $|\rho| < 1$, the spatial error process is stable thus yielding feedback effects that are bounded.

The vector of innovations is defined as:

$$\boldsymbol{\varepsilon}(t) = \mathbf{I}_N \boldsymbol{\eta} + \mathbf{v}(t), \qquad \mathbf{v} \sim \text{iid}(0, \sigma_v^2 \mathbf{I}_N), \tag{6}$$

where $\boldsymbol{\eta}$ is an $N \times 1$ vector representing (unobservable) unit-specific fixed effects, and $\mathbf{v}(t)$ is an $N \times 1$ vector of independently and identically distributed (iid) error terms with variance σ_v^2, which is assumed to be constant across units and time periods. In the following, we focus on a specification in which $\boldsymbol{\eta}$ is correlated with the regressors.

Equations (1), (5), and (6) can be written concisely as:

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{z}(t)\boldsymbol{\theta} + \mathbf{u}(t), && (7) \\ \mathbf{u}(t) &= (\mathbf{I}_N - \rho \mathbf{M}_N)^{-1} [\mathbf{I}_N \boldsymbol{\eta} + \mathbf{v}(t)], && (8) \end{aligned}$$

[11] If $\mathbf{W}_N = \mathbf{M}_N$, then parameters δ and ρ cannot be disentangled when estimated by ML (cf. Anselin, 2006). When using GMM estimation, however, δ and ρ can be separately identified.

where $\mathbf{z}(t) \equiv [\mathbf{y}(t-1), \mathbf{W}_N\mathbf{y}(t), \mathbf{x}(t)]$ denotes the matrix of regressors, $\boldsymbol{\theta} \equiv [\lambda, \delta, \boldsymbol{\beta}']'$ is a vector of $K+2$ parameters, and a prime denotes a transpose. Our general dynamic spatial panel data model embeds various special cases discussed in the literature. If $\lambda = \rho = 0$ and $\delta > 0$, our model reduces to the familiar spatial lag model (also known as the mixed regressive-spatial autoregressive model; see Anselin, 1988), whereas for $\lambda = \rho = \beta = 0$ we get a pure spatial autoregressive model. If $\lambda = \delta = 0$ and $\rho > 0$, we obtain the spatial error model. If $\lambda > 0$ and $\delta = \rho = 0$, we arrive at Arellano and Bond's dynamic panel data model. Finally, the dynamic general spatial panel data model boils down to a standard static panel data model if $\lambda = \delta = \rho = 0$.

3 Spatial Dynamic Panel Estimators

This section develops the spatial dynamic estimators to be used in the Monte Carlo simulations of Section 4. We extend Kapoor, Kelejian, and Prucha's (2007) approach—which explicitly corrects for spatial error correlation—to include both a time lag and a spatial lag of the dependent variable. Because the time lag is endogenous, we apply a panel GMM procedure. We propose sets of instruments for both the time lag and spatial lag of the dependent variable. This procedure yields consistent spatially corrected Arellano-Bond and Blundell-Bond estimators, which are discussed in Sections 3.1 and 3.2, respectively.

3.1 Spatially Corrected Arellano-Bond Estimator

This section extends Arellano and Bond's (1991) dynamic panel data approach to a spatial setting in which both a spatial lag and a spatial error term are present. The spatially corrected Arellano-Bond estimator (denoted by SCAB) will be derived in three stages.

3.1.1 The First Stage

In the first stage, we employ a GMM estimate of $\boldsymbol{\theta}$, which we use to calculate $\hat{\mathbf{u}}(t) = \mathbf{y}(t) - \mathbf{z}(t)\hat{\boldsymbol{\theta}}$. To eliminate $\boldsymbol{\eta}$ from $\boldsymbol{\varepsilon}(t)$, we take first differences of (7) and (8):[12]

$$\begin{aligned} \Delta\mathbf{y}(t) &= \Delta\mathbf{z}(t)\boldsymbol{\theta} + \Delta\mathbf{u}(t), && (9) \\ \Delta\mathbf{u}(t) &= (\mathbf{I}_N - \rho\mathbf{M}_N)^{-1}\Delta\mathbf{v}(t), \qquad \text{for} \quad t = 3, ..., T, && (10) \end{aligned}$$

where $\Delta\mathbf{q}(t) \equiv \mathbf{q}(t) - \mathbf{q}(t-1)$ for $\mathbf{q}(t) = \{\mathbf{y}(t), \mathbf{z}(t), \mathbf{u}(t), \mathbf{v}(t)\}$. Both the time lag and the spatial lag of the dependent variable are endogenous. In addition, the two endogenous regressors are correlated with each other. Therefore, the challenge is to find spatial instruments that are more strongly correlated with the spatial lag than with the time lag of the dependent variable. Conversely, the dynamic instruments need to be more strongly correlated with the time lag than with the spatial lag. The consistency of the panel GMM procedure[13] relies on the existence of an $N(T-2) \times F$ instrument matrix (i.e., $\mathbf{H}_{SAB}$, see below) that satisfies the following F moment conditions: $\mathrm{E}[\mathbf{H}'_{SAB}\Delta\mathbf{u}] = 0$, where we use stacked notation (sorting the data first by time and then by unit, so that the time subscript can be dropped) and $\mathrm{E}[\cdot]$ is the expectations operator.[14] The resulting first-stage spatial Arellano-Bond (SAB) estimator becomes:[15]

$$\hat{\boldsymbol{\theta}}_{SAB} = \left[\Delta\mathbf{z}'\mathbf{H}_{SAB}\mathbf{A}_{SAB}\mathbf{H}'_{SAB}\Delta\mathbf{z}\right]^{-1} \Delta\mathbf{z}'\mathbf{H}_{SAB}\mathbf{A}_{SAB}\mathbf{H}'_{SAB}\Delta\mathbf{y}, \tag{11}$$

[12] Instead of using the differencing transformation to eliminate the unit-specific effects, forward orthogonal deviations can be used. The results for this transformation are available from the authors upon request.

[13] It is straightforward to show for fixed small T and $N \to \infty$ that Assumptions 3.1–3.4 underlying Proposition 3.1 (asymptotic distribution of the GMM estimator) of Hayashi (2000) hold and, hence, that the spatial Arellano-Bond estimator is consistent. Based on Kapoor, Kelejian, and Prucha's (2007) consistency proof, it can be easily shown that the final-stage spatial Arellano-Bond estimator (see Equation (20) below) is also consistent.

[14] Note that we use $T-2$ time periods in the first stage of the modified Kapoor, Kelejian, and Prucha (2007) procedure. One observation is lost due to the first differencing operation and another observation is dropped because of the one-period time lag of the dependent variable. The second stage employs $T-1$, reflecting that the variables are defined in levels.

[15] If the model is just identified, the panel GMM estimator simplifies to an instrumental variables estimator.

where $\mathbf{A}_{SAB} = [\mathbf{H}'_{SAB}\mathbf{G}\mathbf{H}_{SAB}]^{-1}$ is an $F \times F$ matrix of instruments, $\mathbf{G} = \mathbf{I}_N \otimes G_{ij}$ is an $N(T-2) \times N(T-2)$ weighting matrix with elements:[16]

$$G_{ij} \equiv \begin{cases} 2 & \text{if } i = j \\ -1 & \text{if } \ i = j+1 \\ -1 & \text{if } \ j = i+1 \\ 0 & \text{otherwise} \end{cases}, \tag{12}$$

and $\otimes$ denotes the Kronecker product.

Let us now discuss the instrument matrix $\mathbf{H}_{SAB}(t)$, which consists of the set of dynamic instruments for the time lag of the dependent variable and the spatial instruments for the spatial lag. Arellano and Bond (1991) propose to use the *levels* of the dependent variable (i.e., $\mathbf{y}(t-2), ..., \mathbf{y}(1)$) as instruments for the time lag of the dependent variable in *first differences* (i.e., $\Delta\mathbf{y}(t-1)$). Because of time dependency in the model, the instruments are correlated with the time lag of the dependent variable in first differences $\Delta\mathbf{y}(t-1)$, but uncorrelated with the error term in first differences (i.e., $\Delta\mathbf{v}(t)$) as the unit-specific effect is eliminated from the first differenced variable. Recall that $\mathbf{y}(t-2)$ is correlated with $\mathbf{v}(t-2), ..., \mathbf{v}(1)$, but not with $\mathbf{v}(t)$ and $\mathbf{v}(t-1)$. The Arellano-Bond procedure implies the moment conditions

$$\mathrm{E}[\mathbf{y}(t-s)'\Delta\mathbf{v}(t)] = 0, \tag{13}$$

for $t = 3, ..., T$ and $s = 2, ..., T-1$. Equation (13) yields $\Gamma = (T-2)(T-1)/2$ potential instruments.

The set of instruments for the spatial lag is based on a modification of Kelejian and Robinson's (1993) approach. We expand the expected value of the spatial lag and take first differences to arrive at $\mathbf{W}_N\Delta\mathbf{x}(t)$.[17] Finally, we include first differences of exogenous

[16] Arellano and Bond (1991) use (12) to yield a one-step estimator that is asymptotically equivalent to the two-step estimator if the v_{ij}'s are independent and homoskedastic both across units and over time.

[17] Kelejian and Robinson (1993) developed their approach for static panel data models. Given that our model is specified in first differences, the instruments are also defined in first differences.

variables as instruments. The $2K$ additional instruments satisfy the following moment condition:

$$\mathrm{E}[(\mathbf{W}_N\Delta\mathbf{x}(t))'\Delta\mathbf{v}(t)] = 0, \qquad \mathrm{E}[\Delta\mathbf{x}(t)'\Delta\mathbf{v}(t)] = 0, \tag{14}$$

for $t = 3, ..., T$. Under the assumption of strict exogeneity of $\mathbf{x}(t)$, the moment conditions in Equation (14) are always met.

The matrix of instruments is defined as $\mathbf{H}_{SAB}(t) = [\mathbf{y}(t-2), ..., \mathbf{y}(1), \mathbf{W}_N\Delta\mathbf{x}(t), \Delta\mathbf{x}(t)]$. For $T = 5$, for example, the instrument matrix has the following structure:

$$\mathbf{H}_{SAB}(5) = \begin{bmatrix} \mathbf{y}(1) & 0 & 0 & 0 & 0 & 0 & \mathbf{W}_N\Delta\mathbf{x}(3) & \Delta\mathbf{x}(3) \\ 0 & \mathbf{y}(2) & \mathbf{y}(1) & 0 & 0 & 0 & \mathbf{W}_N\Delta\mathbf{x}(4) & \Delta\mathbf{x}(4) \\ 0 & 0 & 0 & \mathbf{y}(3) & \mathbf{y}(2) & \mathbf{y}(1) & \mathbf{W}_N\Delta\mathbf{x}(5) & \Delta\mathbf{x}(5) \end{bmatrix}, \tag{15}$$

where the first row of the matrix consists of the instruments for period 3. Columns (1)–(6) contain the instruments for the time lag of the dependent variable, whereas column (7) depicts the instruments for the spatial lag. Column (8) contains the exogenous variables.

3.1.2 The Second Stage

In the second step, consistent GMM estimates of ρ and σ_v^2 are obtained using $\hat{\mathbf{u}}(t)$ and the modified moment conditions of Kapoor, Kelejian, and Prucha (2007). Because we allow for fixed effects, only the first three of the six moment conditions of Kapoor, Kelejian, and Prucha (2007) are relevant.[18] In stacked format these moment conditions are:

$$\begin{bmatrix} \frac{1}{N(T-2)}\boldsymbol{\varepsilon}'\mathbf{Q}\boldsymbol{\varepsilon} \\ \frac{1}{N(T-2)}\bar{\boldsymbol{\varepsilon}}'\mathbf{Q}\bar{\boldsymbol{\varepsilon}} \\ \frac{1}{N(T-2)}\bar{\boldsymbol{\varepsilon}}'\mathbf{Q}\boldsymbol{\varepsilon} \end{bmatrix} = \begin{bmatrix} \sigma_v^2 \\ \sigma_v^2\frac{1}{N}\mathrm{tr}(\mathbf{M}_N'\mathbf{M}_N) \\ 0 \end{bmatrix}, \tag{16}$$

[18]Kapoor, Kelejian, and Prucha (2007) derive six moment conditions for a static panel data model with random effects.

where tr(.) is the trace of the matrix $\mathbf{M}_N'\mathbf{M}_N$, $\mathbf{Q} \equiv (\mathbf{I}_{T-1} - \mathbf{J}_{T-1}/(T-1)) \otimes \mathbf{I}_N$ is the "within" transformation matrix, $\mathbf{I}_{T-1}$ is an identity matrix of dimension $T-1$, and $\mathbf{J}_{T-1} \equiv \mathbf{e}_{T-1}\mathbf{e}_{T-1}'$ is a $(T-1) \times (T-1)$ matrix of unit elements. The $N(T-1) \times 1$ vectors $\boldsymbol{\varepsilon}$ and $\bar{\boldsymbol{\varepsilon}}$ are:

$$\boldsymbol{\varepsilon} \equiv \mathbf{u} - \rho\bar{\mathbf{u}}, \qquad \bar{\boldsymbol{\varepsilon}} \equiv \bar{\mathbf{u}} - \rho\bar{\bar{\mathbf{u}}}, \tag{17}$$

where $\mathbf{u} = \mathbf{y} - \mathbf{z}\hat{\boldsymbol{\theta}}_{SAB}$, $\bar{\mathbf{u}} = (\mathbf{I}_{T-1} \otimes \mathbf{M}_N)\mathbf{u}$, and $\bar{\bar{\mathbf{u}}} = (\mathbf{I}_{T-1} \otimes \mathbf{M}_N)\bar{\mathbf{u}}$. The estimated value of $\mathbf{u}$ (denoted by $\hat{\mathbf{u}}$) is plugged into $\bar{\mathbf{u}}$ and $\bar{\bar{\mathbf{u}}}$. Using $\hat{\mathbf{u}}$, $\bar{\mathbf{u}}$, and $\bar{\bar{\mathbf{u}}}$ into Equation (17), which in turn is substituted into (16), yields three equations in two unknowns:

$$\frac{\boldsymbol{\Omega}}{N}\begin{bmatrix} \rho \\ \rho^2 \\ \sigma_v^2 \end{bmatrix} = \frac{1}{N}\begin{bmatrix} \frac{1}{T-2}\hat{\mathbf{u}}'\mathbf{Q}\hat{\mathbf{u}} \\ \frac{1}{T-2}\hat{\bar{\mathbf{u}}}'\mathbf{Q}\hat{\bar{\mathbf{u}}} \\ \frac{1}{T-2}\mathbf{u}'\mathbf{Q}\hat{\bar{\mathbf{u}}} \end{bmatrix}, \tag{18}$$

where

$$\boldsymbol{\Omega} \equiv \begin{bmatrix} \frac{2}{T-2}\hat{\mathbf{u}}'\mathbf{Q}\hat{\bar{\mathbf{u}}} & -\frac{1}{T-2}\hat{\bar{\mathbf{u}}}'\mathbf{Q}\hat{\bar{\mathbf{u}}} & 1 \\ \frac{2}{T-2}\hat{\bar{\bar{\mathbf{u}}}}'\mathbf{Q}\hat{\bar{\mathbf{u}}} & -\frac{1}{T-2}\hat{\bar{\bar{\mathbf{u}}}}'\mathbf{Q}\hat{\bar{\bar{\mathbf{u}}}} & \mathrm{tr}(\mathbf{M}_N'\mathbf{M}_N) \\ \frac{2}{T-2}\left[\hat{\mathbf{u}}'\mathbf{Q}\hat{\bar{\bar{\mathbf{u}}}} + \hat{\bar{\mathbf{u}}}'\mathbf{Q}\hat{\bar{\mathbf{u}}}\right] & -\frac{1}{T-2}\hat{\bar{\mathbf{u}}}'\mathbf{Q}\hat{\bar{\bar{\mathbf{u}}}} & 0 \end{bmatrix}.$$

This nonlinear system of equations can be solved to obtain estimates of ρ and σ_v^2.

3.1.3 The Third Stage

In the final stage, the estimate of ρ is used to spatially transform the variables in (7) to yield:

$$\Delta\tilde{\mathbf{y}} = \Delta\tilde{\mathbf{z}}\boldsymbol{\theta}_{SCAB} + \Delta\boldsymbol{\varepsilon}, \tag{19}$$

where $\tilde{\mathbf{p}} = [\mathbf{I}_N - \hat{\rho}\mathbf{M}_N]\mathbf{p}$ for $\mathbf{p} = \{\mathbf{y}, \mathbf{z}\}$. The transformed model is used to derive the final-stage SAB estimator:

$$\hat{\boldsymbol{\theta}}_{SCAB} = \left[\Delta\tilde{\mathbf{z}}'\tilde{\mathbf{H}}_{SAB}\tilde{\mathbf{A}}_{SAB}\tilde{\mathbf{H}}_{SAB}'\Delta\tilde{\mathbf{z}}\right]^{-1}\Delta\tilde{\mathbf{z}}'\tilde{\mathbf{H}}_{SAB}\tilde{\mathbf{A}}_{SAB}\tilde{\mathbf{H}}_{SAB}'\Delta\tilde{\mathbf{y}}, \tag{20}$$

where $\tilde{\mathbf{A}}_{SAB} \equiv \left[\tilde{\mathbf{H}}'_{SAB}\mathbf{G}\tilde{\mathbf{H}}_{SAB}\right]^{-1}$ and $\tilde{\mathbf{H}}_{SAB} \equiv [\mathbf{I}_N - \hat{\rho}\mathbf{M}_N]\mathbf{H}_{SAB}$.

3.2 Spatially Corrected Blundell-Bond Estimator

The standard Arellano-Bond estimator is known to be rather inefficient when instruments are weak because it makes use of information contained in first differences of variables only. To address this shortcoming, the GMM approach of Blundell and Bond (1998)—often referred to as system GMM—employs both variables in levels and in first differences in one model. The spatially corrected variant of Blundell and Bond's estimator (denoted by SCBB) can be derived using the following model:

$$\begin{bmatrix} \mathbf{y}(t) \\ \Delta\mathbf{y}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{z}(t) \\ \Delta\mathbf{z}(t) \end{bmatrix} \boldsymbol{\theta} + \begin{bmatrix} \mathbf{u}(t) \\ \Delta\mathbf{u}(t) \end{bmatrix}, \tag{21}$$

which can be expressed more compactly as:

$$\mathbf{y}_{BB}(t) = \mathbf{z}_{BB}(t)\boldsymbol{\theta} + \mathbf{u}_{BB}(t), \tag{22}$$

where $\mathbf{y}_{BB}(t)$ is a $2N \times 1$ vector and BB denotes Blundell-Bond. The Blundell-Bond model structure doubles the number of observations from $N(T-2)$ to $2N(T-2)$, which increases estimation efficiency. Again, we can use the three-step spatial estimation procedure as set out in Section 3.1. Using stacked notation, in the first step, the spatial Blundell-Bond (SBB) estimator is derived:

$$\hat{\boldsymbol{\theta}}_{SBB} = [\mathbf{z}'_{BB}\mathbf{H}_{SBB}\mathbf{A}_{SBB}\mathbf{H}'_{SBB}\mathbf{z}_{BB}]^{-1}\,\mathbf{z}'_{BB}\mathbf{H}_{SBB}\mathbf{A}_{SBB}\mathbf{H}'_{SBB}\mathbf{y}_{BB}, \tag{23}$$

where $\mathbf{A}_{SBB} = [\mathbf{H}'_{SBB}\mathbf{H}_{SBB}]^{-1}$ and $\mathbf{H}_{SBB}$ is defined as:[19]

$$\mathbf{H}_{SBB} = \begin{bmatrix} \mathbf{H}_D & 0 \\ 0 & \mathbf{H}_L \end{bmatrix}. \tag{24}$$

The upper left block of $\mathbf{H}_{SBB}$, denoted by $\mathbf{H}_{\mathrm{D}}$, contains the instruments for the model in first differences and the lower right block, $\mathbf{H}_{\mathrm{L}}$, includes the instruments for the model in levels. Note that the structure of $\mathbf{H}_{SBB}$ ensures that the instruments intended for the variables defined in levels do not interact with the variables in first differences and vice versa. The instrument matrix $\mathbf{H}_D(t)$ (in unstacked form) is based on the following moment conditions:

$$\mathrm{E}[\mathbf{y}(t-s)'\Delta\mathbf{v}(t)] = 0, \quad \mathrm{E}[(\mathbf{W}_N\mathbf{x})'\Delta\mathbf{v}(t)] = 0, \quad \mathrm{E}[\mathbf{x}(t)'\Delta\mathbf{v}(t)] = 0, \tag{25}$$

while $\mathbf{H}_L(t)$ is based on:

$$\mathrm{E}[\Delta\mathbf{y}(t)'\mathbf{v}(t-s)] = 0, \quad \mathrm{E}[(\mathbf{W}_N\Delta\mathbf{x}(t))'\mathbf{v}(t-s)] = 0, \quad \mathrm{E}[\Delta\mathbf{x}(t)'\mathbf{v}(t-s)] = 0, \tag{26}$$

for $t = 3, ..., T$ and $s = 2, ..., T-1$.

Analogously to Section 3.1, the second stage of the estimation procedure uses $\hat{\mathbf{u}} = \mathbf{y} - \mathbf{z}\hat{\boldsymbol{\theta}}_{SBB}$, where $\hat{\boldsymbol{\theta}}_{SBB}$ is obtained from the first-stage of the spatial GMM procedure and $\mathbf{z}$ now contains only the levels of variables. By plugging $\hat{\mathbf{u}}$ into (17) and using this result into the three moment conditions (16), we can solve the system to arrive at $\hat{\rho}$ and $\hat{\sigma}_v^2$. The value of $\hat{\rho}$ is employed to transform the variables in the third stage.

The estimator of $\boldsymbol{\theta}$ in the final stage is:

$$\hat{\boldsymbol{\theta}}_{SCBB} = \left[\tilde{\mathbf{z}}'_{BB}\tilde{\mathbf{H}}_{SBB}\tilde{\mathbf{A}}_{SBB}\tilde{\mathbf{H}}'_{SBB}\tilde{\mathbf{z}}_{BB}\right]^{-1}\tilde{\mathbf{z}}'_{BB}\tilde{\mathbf{H}}_{SBB}\tilde{\mathbf{A}}_{SBB}\tilde{\mathbf{H}}'_{SBB}\tilde{\mathbf{y}}_{BB}, \tag{27}$$

where $\tilde{\mathbf{p}} = [\mathbf{I}_N - \hat{\rho}\mathbf{M}_N]\mathbf{p}$ for $\mathbf{p} = \{\mathbf{y}_{BB}, \mathbf{z}_{BB}\}$. The instruments of the matrix $\tilde{\mathbf{H}}_{SBB}$ are defined as: $\tilde{\mathbf{H}}_{SBB} = [\mathbf{I}_N - \hat{\rho}\mathbf{M}_N]\mathbf{H}_{BB}$ and $\tilde{\mathbf{A}}_{SBB} = \left[\tilde{\mathbf{H}}'_{SBB}\tilde{\mathbf{H}}_{SBB}\right]^{-1}$.

[19]Based on the line of reasoning of footnote 13, it can be easily shown that for small T and $N \to \infty$ both the first-stage and final-stage spatial Blundell-Bond estimators are consistent.

3.3 Econometric Issues

The number of instruments rises exponentially if T increases. When using the instrument matrix $\mathbf{H}_{SAB}$ in short panels (e.g., $T = 5$), this need not be a problem. Multicollinearity problems, however, arise for large T. Using many instruments may "overfit" instrumented variables (i.e., the instruments fail to remove the endogenous component), thereby biasing estimated coefficients toward those of non-instrumented variables (cf. Roodman, 2009).

To address the explosion in instrument count, we use two methods suggested in the literature. First, we use a "collapsed instrument matrix" (e.g., Beck and Levine, 2004), which groups instruments in smaller sets by horizontally aggregating over the columns of (15). For general T, the matrix corresponding to (15) becomes:

$$\mathbf{H}^{C}_{SAB}(t) = \begin{bmatrix} \mathbf{y}(1) & 0 & 0 & \dots & \mathbf{W}_N\Delta\mathbf{x}(3) & \Delta\mathbf{x}(3) \\ \mathbf{y}(2) & \mathbf{y}(1) & 0 & \dots & \mathbf{W}_N\Delta\mathbf{x}(4) & \Delta\mathbf{x}(4) \\ \mathbf{y}(3) & \mathbf{y}(2) & \mathbf{y}(1) & \dots & \mathbf{W}_N\Delta\mathbf{x}(5) & \Delta\mathbf{x}(5) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \end{bmatrix}. \tag{28}$$

The collapsing procedure reduces the number of instruments for the time lag of the dependent variable from $\Gamma = (T-2)(T-1)/2$ (for the untransformed matrix) to $\Gamma^C = T-2$. Second, we restrict the lag depth of the dynamic instruments, that is, the number of lags. Based on a lag depth of χ imposed on the untransformed matrix, we find $\Gamma^L = \Gamma - (T-2-\chi)(T-1-\chi)/2$ instruments. By combining the two methods, taking $T = 5$ and $\chi = 2$, for example, we obtain:

$$\mathbf{H}^{CL}_{SAB}(5) = \begin{bmatrix} \mathbf{y}(1) & 0 & \mathbf{W}_N\Delta\mathbf{x}(3) & \Delta\mathbf{x}(3) \\ \mathbf{y}(2) & \mathbf{y}(1) & \mathbf{W}_N\Delta\mathbf{x}(4) & \Delta\mathbf{x}(4) \\ \mathbf{y}(3) & \mathbf{y}(2) & \mathbf{W}_N\Delta\mathbf{x}(5) & \Delta\mathbf{x}(5) \end{bmatrix}. \tag{29}$$

In general, the instrument count becomes $\Gamma^{CL} = \chi$, which is linear in the lag depth.

Unfortunately, there are no formal tests or rules of the thumb to determine the optimal number of instruments. We therefore study a number of variants of instrument count reduction methods (Appendix A.1) and pick the one performing best. In the simulations of Section 4, we use a collapsed instrument matrix in combination with lag depth restrictions. The number of lags is set to five. Note that the restriction on lag depth is only binding if we choose $T > 5$ (given that our benchmark scenario will employ $T = 5$).

4 Monte Carlo Simulations

To assess the performance of the estimators presented in Section 3, this section reports a Monte Carlo experiment. The design of the Monte Carlo experiment is discussed first before turning to the results.

4.1 Simulation Design

We report the small sample properties of the estimators using data sets generated based on the dynamic model of Section 2. To this end, we set $T = 5$ and $N = 60$ in the benchmark design. In generating the data, we follow a three-step procedure. First, we generate the vector of covariates, which includes only one exogenous variable. Following Baltagi et al. (2007), the exogenous variable is defined as:

$$\mathbf{x}(t) = \boldsymbol{\varsigma} + \boldsymbol{\chi}(t), \qquad \boldsymbol{\varsigma} \sim \text{iid U}[-7.5, 7.5], \qquad \boldsymbol{\chi} \sim \text{iid U}[-5, 5], \tag{30}$$

where $\boldsymbol{\varsigma}$ represents the unit-specific component and $\boldsymbol{\chi}(t)$ denotes a random component; both are drawn from a uniform distribution, U, defined on a pre-specified interval.

The second step generates the error component $\mathbf{u}(t)$ using:

$$\boldsymbol{\eta} \sim \text{iid U}[-1, 1], \qquad \mathbf{v} \sim \text{iid N}(0, \mathbf{I}_N). \tag{31}$$

The third step generates data on the dependent variable $\mathbf{y}(t)$ and the spatial lag, $\mathbf{W}_N\mathbf{y}(t)$. The data generation process is given by Equations (7)–(8) for $t = 2, ..., T$ and $\mathbf{y}(1) = \boldsymbol{\eta}$. The first $100 - T$ observations of the Monte Carlo runs are discarded to ensure that the results are not unduly affected by the initial values (cf. Hsiao, Pesaran, and Tahmiscioglu, 2002).[20] Following standard practice in the literature, we use different weight matrices for the spatial lag and spatial error component, that is, $\mathbf{W}_N \neq \mathbf{M}_N$. The weight matrices are randomly generated—featuring weights that are kept fixed throughout the simulations—and meet the criteria set out in Section 2.

In the benchmark specification, the parameters in (7)–(8) take on the following values in the data generation process. As is standard practice in the literature, the coefficient of the exogenous explanatory variable β is set to unity. Note that the model does not feature a common intercept across cross-sectional units. The spatial autocorrelation coefficient ρ is set to -0.3 in the simulations. A negative value of ρ implies that an unobserved positive shock in the equation for spatial unit i reduces the dependent variable in other spatial units $i \neq j$. We set $\lambda = 0.3$ and $\delta = 0.5$, so that the stationarity conditions of Section 2 are satisfied. These parameter values yield an average adjusted R^2 of approximately 0.84. Finally, we perform some robustness tests by considering various values of T (ranging from 5 to 50), different values of ρ (ranging from -0.8 to 0.8 to satisfy $|\rho| < 1$), and various values of δ and λ (also ranging -0.8 to 0.8). To stay within the stability bounds, we set $\lambda = 0.1$ if we vary δ. Similarly, we employ $\delta = 0.1$ if we vary λ.

For each experiment, the performance of the estimators is computed based on $1,000$ replications. Following Kapoor, Kelejian, and Prucha (2007) and others, we measure performance by the RMSE $= \sqrt{bias^2 + \left(\frac{q_1 - q_2}{1.35}\right)^2}$, where $bias$ denotes the difference between the median and the "true" value of the parameter of interest (i.e., the value imposed in the data-generating process) and $q_1 - q_2$ is the interquantile range (where q_1 is the 0.75 quantile and q_2 is the 0.25 quantile). If the distribution is normal, $(q_1 - q_2)/1.35$ comes close (aside from a rounding error) to the standard deviation of the estimate.[21] The RMSE

[20] We have checked the robustness of the results with respect to changes in the initial values.

[21] We have used the median instead of the mean in summarizing the distribution because the former is less sensitive to outliers.

thus consists of the bias of the estimator (the first term) and a measure of the distribution of the estimate (the second term).

To verify the validity of the instrument set, we test whether second-order serial correlation is present in the residuals of our model. We also perform a test for first-order serial correlation. Finally, we use the Sargan-Hansen overidentification test, which employs the strict exogeneity of the instruments as null hypothesis. Because neither of these tests produced significant p-values at the 10 percent level—implying that the respective null hypotheses cannot be rejected—we do not report the results.

In the simulations, we compare various estimators with each other. We use four different types of spatial GMM estimators all of which instrument the time lag of the dependent variable (in addition to addressing spatial aspects). The spatially corrected Arellano-Bond estimator (labeled SCAB) and spatially corrected Blundell-Bond estimator (denoted SCBB) correct for spatial error correlation and apply appropriate instruments for the spatial lag of the dependent variables. These two estimators correspond to the final stage of the three-stage spatial GMM procedure (as discussed in Sections 3.1 and 3.2). In addition, we consider a spatial Arellano-Bond estimator (labeled SAB) and a spatial Blundell-Bond estimator (labeled SBB), which instrument the spatial lag of the dependent variable—using a modification of Kelejian and Robinson's (1993) approach—but do not correct for spatial error correlation.[22] These two estimators correspond to the first stage of the three-stage spatial GMM procedure. We compare the spatial GMM estimators with the modified least squares dummy variables (MLSDV) estimator, which applies fixed effects and instruments the spatial lag with spatially weighted exogenous variables. Nickell (1981) shows that the regular LSDV estimator yields biased estimates in the case of dynamic panels because the time lag of the dependent variable is correlated with the error term. Although this bias

[22]We also consider an alternative approach in which the spatial lag of the dependent variable is instrumented with various time lags of the spatially lagged dependent variable in addition to the modified Kelejian and Robinson set of instruments. Appendix A.2 shows that the performance of the estimators does not differ much from that of the benchmark specification.

approaches zero as the number of time periods tends to infinity, it cannot be ignored in small samples.

4.2 Results

Table 1 reports the bias and RMSE in the parameters λ, δ, and β based on 1,000 replications for the five estimators (i.e., MLSDV, SAB, SCAB, SBB, and SCBB) and various values of T starting at the benchmark value $T = 5$. Note that the parameter ρ is only estimated in case of the SCAB en SCBB estimators, but features in the data generation process in all cases. The bias in the coefficient of the lagged dependent variable (λ) is large and negative when using the MLSDV estimator. For large values of T, the bias gets smaller rapidly. The bias in λ is reduced if we use the SAB estimator or its spatially corrected variant. The system-based GMM estimators (SBB and SCBB) result in a substantial reduction in bias. The SBB estimator yields a slightly smaller bias than SCBB in the benchmark case. If T takes on large values, the difference between the SAB and SCAB estimators, on the one hand, and SBB and SCBB estimators, on the other hand, disappears. Not surprisingly, the spatial GMM estimators perform better than the MLSDV estimator across all considered values of T.

The bias in the spatial interaction coefficient δ when using the MLSDV estimator is negative and in absolute terms much smaller than that for λ. Elhorst (2008b) finds a similar result in his Monte Carlo simulations when using the CLSDV estimator. In contrast to the negative bias obtained in δ when using the MLSDV estimator, the bias for the spatial GMM estimators is positive in all cases. In absolute terms, the bias in δ generated by the spatial GMM estimators is smaller than that of the MLSDV estimator. The bias in δ when using the spatial estimators is small; in the benchmark scenario, it amounts on average to 0.50 percent of the true δ. This bias is much smaller than that found by Elhorst

(2008b), who reports a bias of 25.8 percent of δ.[23] Surprisingly, the difference-based GMM estimators yield a smaller bias for small panels than their system-based counterparts. This pattern does not uphold, however, for large values of T.

The MLSDV estimator produces the largest absolute bias in estimating β, but it quickly converges to a negligible value for large values of T. All spatial estimators yield a bias close to zero; in the benchmark scenario, the bias of the SAB estimator is the largest. The differences in bias across estimators are small, however. In estimating ρ in the benchmark scenario, the SCBB estimator produces a smaller bias than the SCAB estimator. When using the SCBB estimator for $T = 5$, the bias in ρ amounts to 1.61 percent of its true value (in absolute terms). Over time, the bias in ρ rises, reaching a peak at $T = 10$, and subsequently shrinks. If we average the absolute bias in ρ over time, it amounts to 0.036 (12 percent of the true parameter value) and 0.021 (7 percent) for SCAB and SCBB, respectively.

The RMSE of the parameters λ, δ, β, and ρ is also presented in Table 1. In line with expectations, the RMSE of each parameter decreases for large values of T. Extending the time period from 5 to 50 reduces the RMSE by 78.5 percent on average (across all parameters and estimators). The drop in RMSE is particularly large when five time periods are added starting from $T = 5$. In the benchmark scenario, the system-GMM based estimators give rise to the smallest RMSE in λ. When estimating δ, we find that the MLSDV estimator yields the smallest RMSE, which is followed by the spatially corrected GMM estimators. A similar pattern is observed for β, but the differences between the spatial estimators get small quickly.

Table 2 shows the performance of the estimators for alternative values of ρ in the interval $[-0.8, 0.8]$ in steps of 0.2. All simulations are based on $T = 5$. For large positive

[23] Elhorst (2008b) uses a GMM approach without spatial error correlation in the data generation process. In addition, he employs a slightly different parameter setting (i.e., $\lambda = 0.4$, $\delta = 0.4$, $\beta = 1$, $N = 60$, and $T = 5$), a different instrument set (without collapsing and lag length restrictions), and a Bucky Ball spatial weight matrix.

values of ρ, across all parameter estimates, we observe an increase in the bias of the SAB and SBB estimators, whereas the bias of the spatially corrected estimators (i.e., SCAB and SCBB) remains stable. The difference in bias at the extreme value of $\rho = 0.8$ amounts to 0.049 (or 6.1 percent of the true parameter value). For large negative values of ρ, the SAB and MBB estimators yield a small bias, however. A similar pattern is visible for the RMSE of the estimators. Note that when looking at λ, δ, and β, the MLSDV estimator produces for all values of ρ the largest bias. Only for very large values of ρ do we observe that the MLSDV estimator generates a smaller bias in δ than the SAB and SBB estimators. The SCBB estimator of ρ yields in most cases the smallest bias. For the case of a pure spatial lag model (i.e., $\rho = 0$), both the MLSDV and spatial estimators reach their lowest average absolute bias. In this case, the MLSDV estimator performs the best in terms of RMSE when estimating δ and the system-based GMM estimators when estimating λ. The spatially corrected GMM estimators produce a large bias and RMSE in estimating $\rho = 0$.

Figure 1 shows the average (absolute) bias and RMSE of our set of estimators for alternative values of λ. Table A2 contains the results for individual parameters. For models with a large λ (which generates strong time dependency), the system-based GMM estimators perform much better than their difference-based GMM counterparts. As expected, the average absolute bias and RMSE of the MLSDV estimator increases with λ. For negative and small positive values of λ, the average absolute bias of the spatial estimators is small and hardly differs across estimators. Note that the bias of the SCAB and SCBB estimators in estimating ρ (not shown in the figure) fluctuates quite a bit; it reaches its largest value for positive values of λ (Table A2). In estimating ρ, the difference between system-based GMM estimators and difference-based GMM estimators is negligible.

The average (absolute) bias of the spatial estimators is small across all values of δ (Figure 2). On average, the system-based GMM estimators yield a smaller average absolute bias than the difference-based GMM estimators. Whether or not a spatial lag is included in the model does not seem to produce a discernable effect on the average absolute bias of the spatial estimators. Compared to the spatial estimators, the MLSDV estimator has a relatively large bias, which is primarily caused by the large bias in λ. The RMSE of the MLSDV estimator is smaller than the difference-based estimators; the system-based GMM estimators feature the lowest RMSE. The RMSE of the MLSDV estimator falls for large positive values of δ in the data generating process. Again, the bias and RMSE of the SCAB and SCBB estimators when estimating ρ (not shown in the figure) fluctuates quite a bit (Table A3).

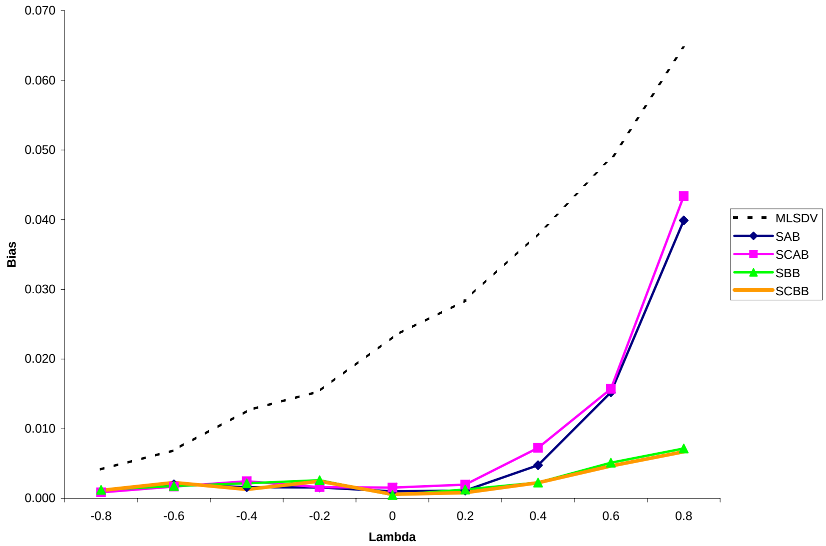
5 Conclusion

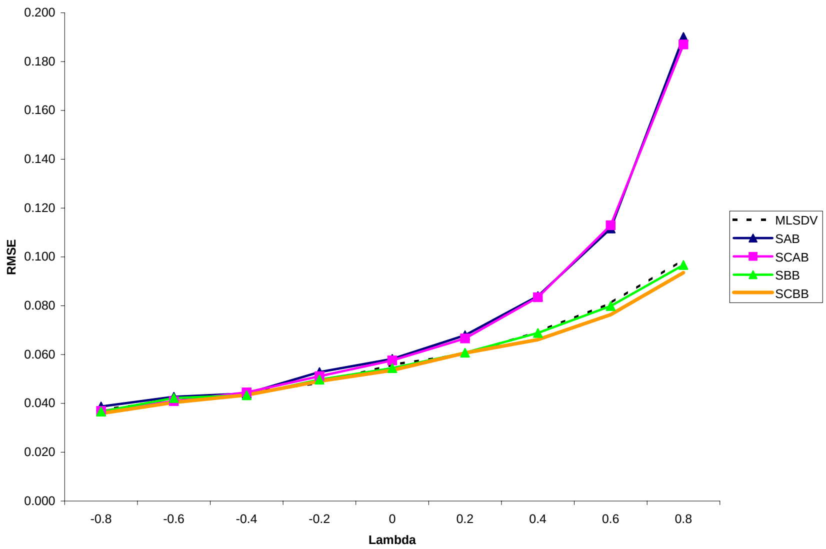
This paper has dealt with Generalized Methods of Moments (GMM) estimation of spatial dynamic panel data models with spatially correlated errors. We extended the three-step GMM approach of Kapoor, Kelejian, and Prucha (2007), which corrects for spatially correlated errors in static panel data models, by introducing a spatial lag and a one-period lag of the endogenous variable as additional explanatory variables. Combining the extended Kapoor, Kelejian, and Prucha (2007) approach with the dynamic panel data model GMM estimators of Arellano and Bond (1991) and Blundell and Bond (1998) and supplementing the dynamic instruments by weighted exogenous variables as suggested by Kelejian and Robinson (1993) yielded new spatial dynamic panel data estimators.

Monte Carlo simulations indicated that differences in bias as well as root mean squared error between spatially corrected GMM estimates and corresponding GMM estimates in which spatial error correlation is ignored are small for reasonable parameter values. The average absolute bias and root mean squared error of the spatial GMM estimators is not affected much by the size of the spatial lag parameter in the data generating process. Choosing appropriate instruments for both the time and spatial lag is thus sufficient to model the spatial characteristics in our setting. We showed that the combination of collapsing the instrument matrix and limiting the lag depth of the dynamic instruments substantially reduces the bias in estimating the spatial lag parameter, but hardly affects its root mean squared error.

In future research, we will investigate theoretically the asymptotic properties of our spatial GMM estimators. Both the dynamic Arellano-Bond estimator and the static spatial GMM estimator of Kelejian and Robinson (1993) are consistent. What remains to be proved is whether the estimators are consistent for the cases of: (i) an infinitely large number of time periods relative to a fixed number of units; and (ii) an infinitely large number of time periods and units. In addition, we intend to apply our estimator to spatial models with endogenous interaction effects. For example, governments competing with each other in setting their corporate tax rates or expenditure levels. Finally, we will add a spatially weighted time lag to the model and investigate the consequences of replacing spatial error correlation by spatially weighted covariates in the model.

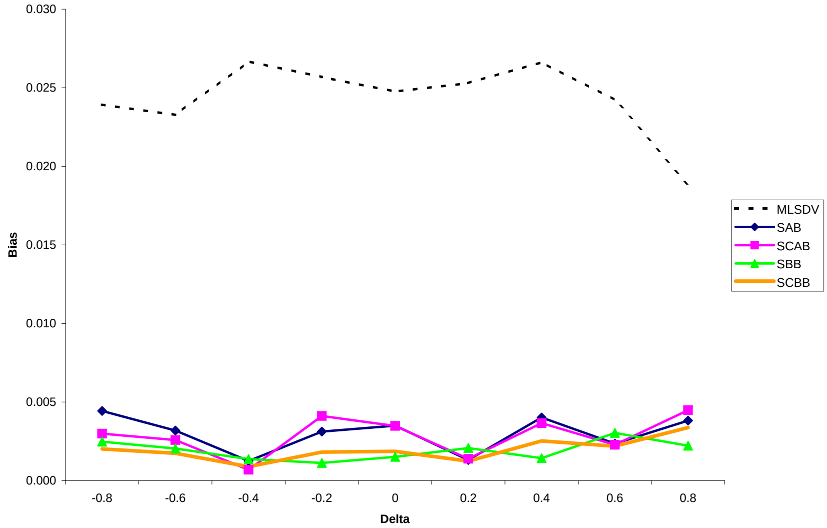
Figure 1: Bias and RMSE of various spatial GMM estimators for different values of λ

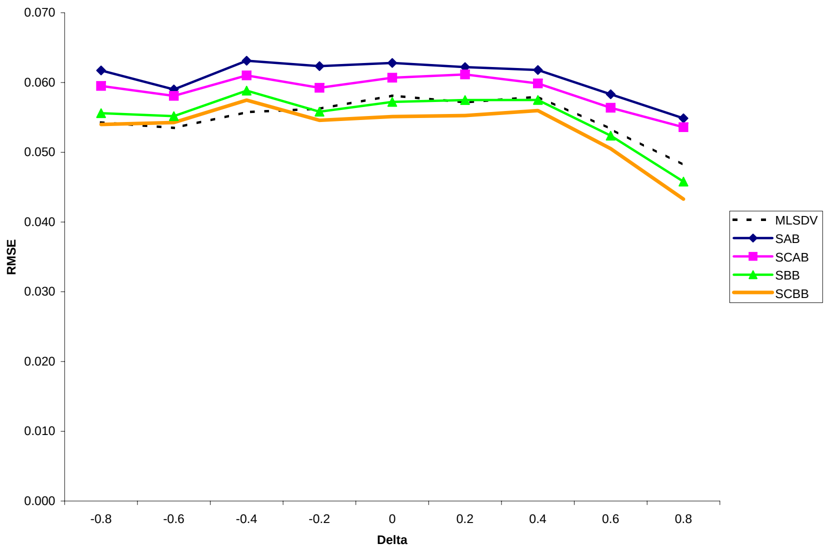




Notes: The absolute bias and RMSE are averaged over λ, δ, and β. Based on: $N = 60$, $\delta = 0.1$, $\beta = 1$, and $\rho = -0.3$. See Table A.2 for further details.

Figure 2: Bias and RMSE of various spatial GMM estimators for different values of δ





Notes: The absolute bias and RMSE are averaged over λ, δ, and β. Based on: $N = 60$, $\lambda = 0.1$, $\beta = 1$, and $\rho = -0.3$. See Table A.3 for further details.

Table 1: Bias and RMSE of various spatial GMM estimators for different values of T

Estimator	Parameter	bias/RMSE	5	10	15	20	50
MLSDV	λ	bias	-0.070	-0.028	-0.017	-0.012	-0.005
		RMSE	0.079	0.035	0.022	0.018	0.009
	δ	bias	-0.013	-0.005	-0.001	-0.001	0.000
		RMSE	0.069	0.038	0.031	0.025	0.015
	β	bias	-0.019	-0.005	-0.002	-0.001	0.000
		RMSE	0.044	0.023	0.018	0.016	0.010
SAB	λ	bias	-0.011	-0.003	0.001	-0.001	0.000
		RMSE	0.083	0.040	0.026	0.021	0.011
	δ	bias	0.001	0.001	0.003	0.002	0.000
		RMSE	0.083	0.050	0.042	0.034	0.020
	β	bias	-0.002	-0.001	0.001	-0.001	0.000
		RMSE	0.057	0.030	0.022	0.019	0.011
SCAB	λ	bias	-0.011	-0.003	0.000	-0.002	0.000
		RMSE	0.085	0.037	0.026	0.022	0.011
	δ	bias	0.000	0.003	0.002	0.002	0.001
		RMSE	0.079	0.047	0.040	0.032	0.019
	β	bias	0.000	-0.001	0.000	-0.001	0.000
		RMSE	0.058	0.030	0.023	0.018	0.011
	ρ	bias	-0.084	0.057	0.017	0.006	0.017
		RMSE	0.155	0.093	0.062	0.050	0.037
SBB	λ	bias	-0.003	-0.001	0.000	-0.001	0.001
		RMSE	0.050	0.032	0.022	0.020	0.011
	δ	bias	0.005	0.001	0.003	0.001	0.001
		RMSE	0.083	0.053	0.046	0.037	0.021
	β	bias	0.000	-0.001	0.001	0.000	0.000
		RMSE	0.049	0.028	0.022	0.018	0.012
SCBB	λ	bias	-0.002	-0.002	0.000	-0.001	0.000
		RMSE	0.050	0.032	0.022	0.019	0.010
	δ	bias	0.003	0.001	0.002	0.001	0.001
		RMSE	0.075	0.049	0.042	0.034	0.020
	β	bias	0.000	-0.001	0.000	0.000	-0.001
		RMSE	0.046	0.028	0.023	0.018	0.011
	ρ	bias	0.005	0.063	0.017	-0.004	-0.017
		RMSE	0.128	0.096	0.063	0.050	0.035

Notes: Based on Monte Carlo simulations with 1,000 replications. The collapsed instrument matrix is employed; the lag depth of the instruments is restricted to five periods. The parameters in the benchmark scenario are: $N = 60$, $\lambda = 0.3$, $\delta = 0.5$, $\beta = 1$, and $\rho = -0.3$. The labels MLSDV, SAB, SCAB, SBB, and SCBB denote modified least squares dummy variables, spatial Arellano-Bond, spatially corrected Arellano-Bond, spatial Blundell-Bond, and spatially corrected Blundell-Bond, respectively.

Table 2: Bias and RMSE of various spatial GMM estimators for different values of ρ

Estimator	Parameter	bias/RMSE	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
MLSDV	λ	bias	-0.082	-0.074	-0.069	-0.067	-0.064	-0.068	-0.071	-0.084	-0.115
		RMSE	0.093	0.083	0.077	0.075	0.072	0.076	0.081	0.094	0.136
	δ	bias	-0.016	-0.012	-0.011	-0.014	-0.012	-0.015	-0.017	-0.017	-0.022
		RMSE	0.077	0.073	0.068	0.067	0.064	0.068	0.072	0.077	0.111
	β	bias	-0.023	-0.026	-0.024	-0.019	-0.020	-0.020	-0.023	-0.026	-0.033
		RMSE	0.049	0.049	0.046	0.045	0.046	0.042	0.043	0.050	0.062
SAB	λ	bias	-0.010	-0.009	0.003	-0.009	-0.003	-0.011	-0.014	-0.014	-0.047
		RMSE	0.093	0.089	0.081	0.086	0.075	0.084	0.101	0.108	0.146
	δ	bias	0.001	0.003	0.002	0.006	0.001	0.001	0.005	0.013	0.066
		RMSE	0.090	0.088	0.089	0.082	0.081	0.082	0.089	0.097	0.158
	β	bias	-0.003	-0.005	0.001	-0.003	-0.002	-0.002	-0.011	-0.004	-0.018
		RMSE	0.064	0.062	0.057	0.056	0.056	0.054	0.058	0.070	0.093
SCAB	λ	bias	-0.010	-0.009	0.000	-0.009	-0.002	-0.010	-0.010	-0.001	-0.007
		RMSE	0.081	0.081	0.083	0.084	0.076	0.080	0.090	0.080	0.084
	δ	bias	0.001	0.007	0.004	0.006	0.000	0.001	0.002	0.005	0.006
		RMSE	0.066	0.080	0.085	0.083	0.082	0.082	0.081	0.079	0.082
	β	bias	-0.002	-0.006	-0.002	-0.003	-0.001	-0.004	-0.008	0.001	-0.003
		RMSE	0.055	0.057	0.060	0.056	0.057	0.052	0.053	0.054	0.052
	ρ	bias	-0.173	0.100	-0.013	0.070	0.129	0.065	-0.049	-0.057	-0.023
		RMSE	0.218	0.167	0.126	0.149	0.183	0.137	0.120	0.119	0.113
SBB	λ	bias	-0.004	-0.005	0.002	-0.003	0.000	-0.006	-0.003	-0.006	-0.023
		RMSE	0.060	0.057	0.053	0.050	0.047	0.054	0.054	0.064	0.097
	δ	bias	0.006	0.005	0.007	0.002	0.003	0.004	0.004	0.015	0.065
		RMSE	0.095	0.095	0.089	0.081	0.080	0.083	0.090	0.094	0.142
	β	bias	0.000	-0.001	0.000	0.002	0.000	-0.001	-0.005	0.000	-0.013
		RMSE	0.055	0.051	0.049	0.045	0.047	0.041	0.046	0.056	0.079
SCBB	λ	bias	-0.002	-0.003	0.001	-0.001	0.001	-0.005	0.000	-0.002	-0.001
		RMSE	0.047	0.048	0.049	0.049	0.048	0.053	0.050	0.052	0.051
	δ	bias	0.000	0.006	0.009	0.004	0.002	0.001	0.001	0.004	0.008
		RMSE	0.062	0.076	0.082	0.079	0.083	0.086	0.084	0.082	0.084
	β	bias	0.000	-0.001	0.000	0.002	0.000	-0.001	-0.002	0.003	0.000
		RMSE	0.045	0.042	0.047	0.047	0.046	0.040	0.044	0.044	0.042
	ρ	bias	-0.188	0.081	-0.021	0.057	0.120	0.071	-0.043	-0.046	-0.001
		RMSE	0.230	0.150	0.130	0.138	0.174	0.140	0.122	0.111	0.102

Notes: Based on Monte Carlo simulations with 1,000 replications. The collapsed instrument matrix is used. The parameters in the benchmark scenario are: $N = 60$, $\lambda = 0.3$, $\delta = 0.5$, and $\beta = 1$. The labels MLSDV, SAB, SCAB, SBB, and SCBB denote modified least squares dummy variables, spatial Arellano-Bond, spatially corrected Arellano-Bond, spatial Blundell-Bond, and spatially corrected Blundell-Bond, respectively.

Appendix

A.1 Reducing the Instrument Count

To study the effects on parameter bias and RMSE of a reduction in instrument count, we focus (for simplicity) on one parameter and estimator. For this purpose, we pick δ and the SAB estimator. We consider five SAB variants. SAB1 uses the full instrument set without any collapsing. SAB2 employs an uncollapsed matrix and restricts the lag length to 10 periods. SAB3 shortens the lag length further to five periods. SAB4 collapses the instrument matrix and constrains the lag length to 10 time periods. SAB5 restricts the lag length to five periods in a collapsed matrix format.

The results in Figure A1 show that SAB5 yields the smallest bias, but does not yield the smallest RMSE at small and intermediate values of T. The difference in RMSEs, however, is small; the estimators based on the uncollapsed instrument matrices seem to be performing a bit better than their collapsed counterparts. The results for SAB1 and SAB2 show an explosion in bias over time. Restricting the lag length to five periods in SAB3 brings the bias down substantially. The collapsed instrument matrices yield a smaller bias than the uncollapsed ones. The collapsed matrix with a lag length of five periods has the smallest bias.

A.2 Introducing Dynamic Instruments for the Spatial Lag

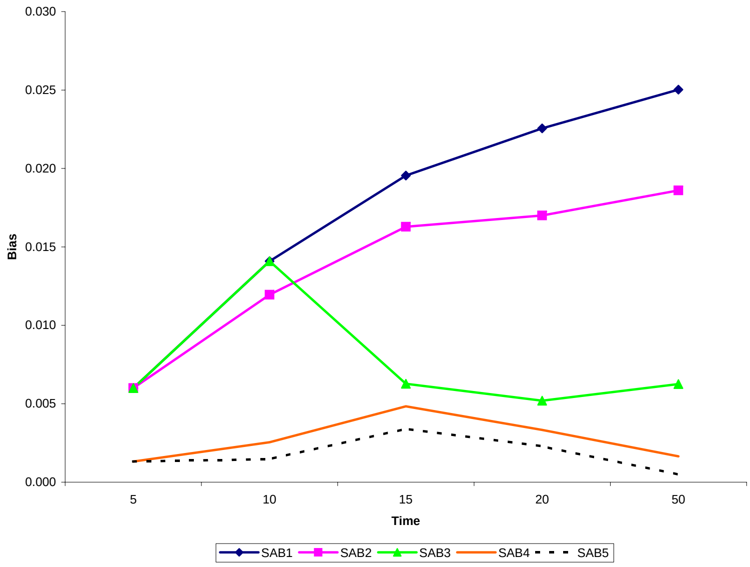
We introduce an additional set of dynamic instruments for the spatial lag, which is based on various time lags of the spatially lagged dependent variable. For $T = 5$, for example, we find the following matrix of instruments for the spatial Arellano-Bond estimator:

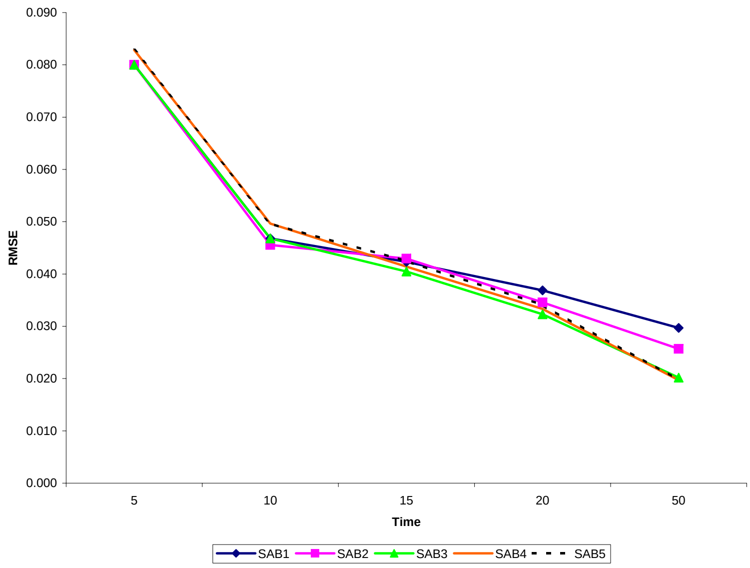
$$\bar{\mathbf{H}}_{SAB}(5) = \begin{bmatrix} \mathbf{y}(1) & 0 & 0 & \mathbf{W}_N\mathbf{y}(1) & 0 & 0 & \mathbf{W}_N\Delta\mathbf{x}(4) & \Delta\mathbf{x}(4) \\ \mathbf{y}(2) & \mathbf{y}(1) & 0 & \mathbf{W}_N\mathbf{y}(2) & \mathbf{W}_N\mathbf{y}(1) & 0 & \mathbf{W}_N\Delta\mathbf{x}(5) & \Delta\mathbf{x}(5) \\ \mathbf{y}(3) & \mathbf{y}(2) & \mathbf{y}(1) & \mathbf{W}_N\mathbf{y}(3) & \mathbf{W}_N\mathbf{y}(2) & \mathbf{W}_N\mathbf{y}(1) & \mathbf{W}_N\Delta\mathbf{x}(6) & \Delta\mathbf{x}(6) \end{bmatrix}.$$

The two instrument blocks of $\bar{\mathbf{H}}_{SAB}$ differ from the instruments in $\mathbf{H}_{SAB}$ in two ways. First of all, the blocks of $\bar{\mathbf{H}}_{SAB}$ include various time lags of the spatially lagged dependent variable as instruments for the spatial lag (columns 4–6) in addition to the modified Kelejian Robinson instruments (column 7). The standard dynamic instruments for the time lag of the dependent variable are included in columns 1–3. Second, we use a collapsed instrument matrix according to the collapsing procedure discussed in the main text.

The average absolute bias (see Table A1) across all estimators is small and shows a pattern very similar to the difference between spatially correlated estimates and their uncorrected counterparts as set out in Table 2. Introducing spatially weighted lagged dependent variables increases the average bias somewhat. Taking averages across all estimates (in absolute terms), yields a bias of 0.016 compared to 0.013 in Table 2. The average RMSE across all parameters and estimators does not differ much from that of the specifications without the dynamic instruments for the spatial lag; it amounts to 0.075 compared to 0.08 in Table 2.

Figure A1: Reducing the Instrument Count





Notes: Bias and RMSE of five variants of SAB estimators for δ. SAB1 (uncollapsed and no lag restrictions), SAB2 (uncollapsed and 10 lags only), SAB3 (uncollapsed and five lags only), SAB4 (collapsed and 10 lags only), and SAB5 (collapsed and five lags only).

Table A1: Bias and RMSE of various spatial GMM estimators with spatial instruments for different values of ρ

Estimator	Parameter	bias/RMSE	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
MLSDV	λ	bias	-0.082	-0.074	-0.069	-0.067	-0.064	-0.068	-0.071	-0.084	-0.115
		RMSE	0.093	0.083	0.077	0.075	0.072	0.076	0.081	0.094	0.136
	δ	bias	-0.016	-0.012	-0.011	-0.014	-0.012	-0.015	-0.017	-0.017	-0.022
		RMSE	0.077	0.073	0.068	0.067	0.064	0.068	0.072	0.077	0.111
	β	bias	-0.023	-0.026	-0.024	-0.019	-0.020	-0.020	-0.023	-0.026	-0.033
		RMSE	0.049	0.049	0.046	0.045	0.046	0.042	0.043	0.050	0.062
ESAB	λ	bias	-0.017	-0.014	-0.005	-0.013	-0.010	-0.018	-0.017	-0.026	-0.074
		RMSE	0.087	0.085	0.076	0.081	0.072	0.078	0.087	0.091	0.139
	δ	bias	0.000	0.004	0.004	0.005	0.002	0.009	0.011	0.031	0.120
		RMSE	0.083	0.076	0.080	0.076	0.074	0.076	0.086	0.099	0.183
	β	bias	-0.003	-0.007	-0.003	-0.005	-0.004	-0.006	-0.013	-0.009	-0.029
		RMSE	0.063	0.060	0.056	0.054	0.054	0.051	0.055	0.062	0.080
ESCAB	λ	bias	-0.015	-0.014	-0.005	-0.015	-0.011	-0.017	-0.013	-0.006	-0.016
		RMSE	0.069	0.074	0.070	0.079	0.073	0.075	0.083	0.076	0.076
	δ	bias	0.001	0.007	0.008	0.005	0.003	0.005	-0.002	0.006	0.010
		RMSE	0.063	0.069	0.076	0.074	0.074	0.080	0.080	0.079	0.076
	β	bias	-0.004	-0.009	-0.004	-0.005	-0.003	-0.006	-0.010	-0.001	-0.007
		RMSE	0.051	0.051	0.053	0.054	0.054	0.052	0.050	0.052	0.052
	ρ	bias	-0.181	0.089	-0.021	0.059	0.122	0.061	-0.050	-0.059	-0.038
		RMSE	0.224	0.155	0.127	0.140	0.179	0.134	0.124	0.119	0.124
ESBB	λ	bias	-0.006	-0.007	-0.001	-0.004	-0.001	-0.008	-0.005	-0.012	-0.037
		RMSE	0.058	0.055	0.050	0.049	0.045	0.051	0.050	0.058	0.087
	δ	bias	0.001	0.005	0.006	0.004	0.002	0.006	0.008	0.024	0.095
		RMSE	0.075	0.072	0.070	0.065	0.064	0.066	0.071	0.083	0.154
	β	bias	0.001	-0.003	-0.002	0.003	-0.001	-0.002	-0.006	-0.006	-0.018
		RMSE	0.053	0.049	0.047	0.046	0.045	0.040	0.043	0.051	0.067
ESCBB	λ	bias	-0.004	-0.006	-0.001	-0.003	-0.001	-0.008	-0.001	-0.004	-0.003
		RMSE	0.041	0.043	0.045	0.046	0.045	0.051	0.047	0.050	0.047
	δ	bias	0.000	0.007	0.005	0.002	0.001	0.004	-0.001	0.005	0.008
		RMSE	0.051	0.057	0.061	0.061	0.066	0.068	0.067	0.069	0.066
	β	bias	0.000	-0.003	-0.002	0.002	-0.001	-0.002	-0.002	0.001	-0.001
		RMSE	0.040	0.039	0.043	0.046	0.044	0.040	0.040	0.042	0.041
	ρ	bias	-0.202	0.072	-0.030	0.054	0.119	0.070	-0.042	-0.045	-0.006
		RMSE	0.239	0.143	0.133	0.135	0.170	0.138	0.121	0.112	0.105

Notes: Based on Monte Carlo simulations with 1,000 replications. The parameters in the benchmark scenario are: $N = 60$, $\lambda = 0.1$, $\delta = 0.5$, and $\beta = 1$. The label E refers to the spatial weighting of lagged dependent variables. The labels MLSDV, SAB, SCAB, SBB, and SCBB denote modified least squares dummy variables, spatial Arellano-Bond, spatially corrected Arellano-Bond, spatial Blundell-Bond, and spatially corrected Blundell-Bond, respectively.

Table A2: Bias and RMSE of various spatial GMM estimators for different values of λ

Estimator	Parameter	bias/RMSE	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
MLSDV	λ	bias	-0.009	-0.017	-0.028	-0.039	-0.054	-0.068	-0.081	-0.099	-0.127
		RMSE	0.022	0.029	0.040	0.050	0.063	0.075	0.089	0.106	0.133
	δ	bias	0.001	0.001	0.004	0.001	-0.001	0.001	-0.006	-0.005	-0.012
		RMSE	0.054	0.056	0.056	0.058	0.064	0.066	0.071	0.079	0.096
	β	bias	-0.002	-0.003	-0.006	-0.007	-0.014	-0.017	-0.026	-0.043	-0.055
		RMSE	0.037	0.038	0.038	0.037	0.040	0.040	0.048	0.058	0.067
SAB	λ	bias	0.000	0.000	0.000	-0.001	-0.003	0.001	-0.011	-0.026	-0.078
		RMSE	0.020	0.028	0.038	0.047	0.058	0.071	0.097	0.151	0.293
	δ	bias	0.001	0.003	0.003	0.000	-0.001	0.000	0.000	0.001	0.000
		RMSE	0.054	0.057	0.053	0.064	0.068	0.079	0.093	0.095	0.132
	β	bias	-0.002	0.002	-0.001	0.004	-0.001	0.002	-0.004	-0.019	-0.042
		RMSE	0.043	0.043	0.041	0.048	0.049	0.054	0.063	0.088	0.145
SCAB	λ	bias	0.000	0.001	-0.001	0.001	-0.002	-0.002	-0.011	-0.027	-0.080
		RMSE	0.020	0.028	0.038	0.045	0.056	0.070	0.100	0.155	0.293
	δ	bias	0.001	0.004	0.005	0.001	-0.001	0.002	-0.005	-0.001	-0.008
		RMSE	0.050	0.053	0.053	0.063	0.070	0.079	0.088	0.096	0.125
	β	bias	-0.001	0.000	-0.002	0.003	-0.002	0.002	-0.005	-0.019	-0.042
		RMSE	0.041	0.042	0.043	0.046	0.047	0.050	0.063	0.089	0.144
	ρ	bias	-0.102	-0.075	0.037	0.097	-0.095	0.335	0.013	-0.058	0.095
		RMSE	0.161	0.155	0.126	0.161	0.155	0.358	0.130	0.142	0.170
SBB	λ	bias	0.001	0.001	-0.001	0.001	0.001	0.000	-0.003	-0.003	-0.011
		RMSE	0.020	0.028	0.034	0.041	0.048	0.050	0.059	0.071	0.082
	δ	bias	0.002	0.003	0.005	0.001	0.000	0.000	-0.003	0.007	0.007
		RMSE	0.047	0.054	0.052	0.063	0.073	0.084	0.101	0.115	0.143
	β	bias	-0.001	0.002	-0.001	0.005	0.000	0.003	0.001	-0.005	-0.003
		RMSE	0.043	0.045	0.044	0.046	0.042	0.048	0.047	0.053	0.064
SCBB	λ	bias	0.001	0.002	0.000	0.002	0.000	0.000	-0.002	-0.004	-0.012
		RMSE	0.020	0.027	0.034	0.040	0.047	0.050	0.057	0.069	0.083
	δ	bias	0.002	0.004	0.003	0.001	0.001	0.000	-0.003	0.007	0.004
		RMSE	0.046	0.052	0.053	0.062	0.071	0.085	0.098	0.110	0.136
	β	bias	-0.001	0.002	-0.001	0.005	0.000	0.002	-0.001	-0.003	-0.004
		RMSE	0.042	0.042	0.043	0.045	0.043	0.046	0.043	0.051	0.061
	ρ	bias	-0.053	-0.074	0.027	0.090	-0.085	0.327	-0.005	-0.046	0.136
		RMSE	0.134	0.153	0.125	0.159	0.146	0.349	0.127	0.134	0.192

Notes: Based on Monte Carlo simulations with 1,000 replications. The collapsed instrument matrix is used. The parameters in the benchmark scenario are: $N = 60$, $\delta = 0.1$, $\beta = 1$, and $\rho = -0.3$. The labels MLSDV, SAB, SCAB, SBB, and SCBB denote modified least squares dummy variables, spatial Arellano-Bond, spatially corrected Arellano-Bond, spatial Blundell-Bond, and spatially corrected Blundell-Bond, respectively.

Table A3: Bias and RMSE of various spatial GMM estimators for different values of δ

Estimator	Parameter	bias/RMSE	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
MLSDV	λ	bias	-0.049	-0.052	-0.055	-0.059	-0.059	-0.058	-0.059	-0.052	-0.038
		RMSE	0.058	0.062	0.066	0.068	0.069	0.069	0.068	0.061	0.047
	δ	bias	0.010	0.005	0.008	0.000	-0.001	-0.001	-0.008	-0.008	-0.007
		RMSE	0.065	0.060	0.060	0.060	0.065	0.060	0.064	0.058	0.055
	β	bias	-0.013	-0.013	-0.016	-0.017	-0.014	-0.016	-0.013	-0.013	-0.011
		RMSE	0.040	0.038	0.042	0.041	0.041	0.043	0.042	0.041	0.042
SAB	λ	bias	-0.007	-0.005	0.001	-0.006	-0.005	0.002	-0.008	-0.006	-0.005
		RMSE	0.060	0.061	0.061	0.066	0.063	0.065	0.063	0.059	0.052
	δ	bias	-0.003	-0.004	0.002	-0.001	-0.001	-0.001	-0.002	-0.001	0.006
		RMSE	0.079	0.071	0.078	0.068	0.074	0.073	0.072	0.068	0.060
	β	bias	-0.003	-0.001	-0.001	-0.003	-0.004	-0.001	-0.002	0.000	0.000
		RMSE	0.046	0.045	0.051	0.052	0.051	0.048	0.050	0.048	0.053
SCAB	λ	bias	-0.005	-0.003	0.000	-0.006	-0.002	0.001	-0.008	-0.005	-0.006
		RMSE	0.059	0.059	0.059	0.062	0.061	0.064	0.061	0.058	0.051
	δ	bias	0.000	-0.004	0.001	-0.003	-0.004	-0.002	-0.002	-0.002	0.007
		RMSE	0.075	0.069	0.073	0.065	0.071	0.072	0.068	0.063	0.056
	β	bias	-0.004	0.000	0.000	-0.003	-0.004	0.001	-0.001	0.000	0.000
		RMSE	0.045	0.046	0.051	0.051	0.049	0.048	0.051	0.048	0.054
	ρ	bias	-0.079	0.016	0.133	0.137	-0.056	-0.128	0.087	-0.050	0.089
		RMSE	0.155	0.130	0.178	0.188	0.135	0.180	0.160	0.128	0.167
SBB	λ	bias	-0.003	0.001	0.002	-0.001	0.000	0.003	-0.002	-0.004	-0.002
		RMSE	0.045	0.049	0.050	0.051	0.049	0.047	0.049	0.044	0.036
	δ	bias	-0.002	-0.003	0.000	-0.002	-0.004	-0.003	-0.001	-0.003	0.004
		RMSE	0.079	0.075	0.080	0.071	0.076	0.079	0.075	0.069	0.053
	β	bias	-0.003	0.002	0.002	0.000	0.000	-0.001	0.001	0.002	-0.001
		RMSE	0.043	0.042	0.046	0.046	0.047	0.047	0.048	0.043	0.048
SCBB	λ	bias	-0.002	0.000	0.001	-0.002	-0.001	0.002	-0.005	-0.004	-0.004
		RMSE	0.044	0.047	0.048	0.048	0.049	0.046	0.048	0.043	0.035
	δ	bias	0.000	-0.002	0.000	-0.002	-0.003	-0.001	-0.002	-0.002	0.005
		RMSE	0.076	0.074	0.078	0.069	0.072	0.076	0.074	0.064	0.048
	β	bias	-0.004	0.003	0.002	-0.001	-0.002	-0.001	0.001	0.001	-0.002
		RMSE	0.042	0.042	0.046	0.047	0.045	0.043	0.046	0.044	0.047
	ρ	bias	-0.079	0.017	0.132	0.137	-0.055	-0.129	0.085	-0.053	0.081
		RMSE	0.152	0.130	0.177	0.188	0.133	0.181	0.157	0.129	0.155

Notes: Based on Monte Carlo simulations with 1,000 replications. The collapsed instrument matrix is used. The parameters in the benchmark scenario are: $N = 60$, $\lambda = 0.1$, $\beta = 1$, and $\rho = -0.3$. The labels MLSDV, SAB, SCAB, SBB, and SCBB denote modified least squares dummy variables, spatial Arellano-Bond, spatially corrected Arellano-Bond, spatial Blundell-Bond, and spatially corrected Blundell-Bond, respectively.

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